HW SOL 7.3

March 5, 2018 1:50 PM

Name:_________________________ **Date:**_________________________

Math 10 Honours section 7.3 Probability with Independent and Dependent Events

1. Which of the following are independent or dependent events?

i) Being good in math and being tall

Independent.

ii) Smoking and getting lung cancer

Dependent.

iii) Having a high IQ and being good in Math

Dependent

iv) Flipping a coin, getting a heads first and then getting heads again

Independent → ~~each~~ each flip is independent

v) Flipping a coin 5 times and getting 5 heads in a row, then getting a head again on the next flip

Independent.

vi) Rolling a dice twice, getting a 3 on the first roll and getting a sum of 8

Independent.

2. A coin is tossed 3 times, event "A" is getting a heads on the first toss, event "B" is getting a heads on the second toss, and event "C" is getting two heads. $P(A)=\frac{1}{2}$ $P(B)=\frac{1}{2}$ $P(C)=\frac{3}{8}$.

HHT
HTH
THH

a. Are events "A" and "B" independent?

Yes

b. Are events "A" and "C" independent?

$P(A \& C)=\frac{2}{8}$ $P(A)\times P(C)=\frac{1}{2}\times\frac{3}{8}\neq\frac{1}{4}$. Dependent.

c. Are events "B" and "C" independent?

$P(B \& C)=\frac{2}{8}$ $P(B)\times P(C)=\frac{1}{2}\times\frac{3}{8}\neq\frac{1}{4}$ Dependent.

3. Classify the following events as being mutually exclusive, not mutually exclusive, independent or dependant.

a) Drawing a red card or a spade

Mutually Exclusive

b) Drawing two aces from a deck of cards without replacement.

Dependent.

c) Flipping a coin twice and getting heads both times.

Independent

d) Drawing a face card or a jack.

Not Mutually Exclusive

4. Indicate whether event "A" and "B" would be independent or dependent: If $P(A) = 0.70$, $P(B) = 0.8$, and $P(A \text{ and } B) = 0.56$

SINCE $P(A) \times P(B) = P(A \& B) \longrightarrow$ INDEPENDENT.

5. If the probability that event "A" occurs is 0.2, event "B" is 0.5, and the event of "A and B" is 0.3, then would events "A" and "B" be independent? Explain?

$P(A) \times P(B) \neq P(A \& B)$ $\longleftarrow$ THERE'S DEPENDENCY

$0.2 \times 0.5 \neq 0.3.$

6. Tina and Brenda are two members of the Terry Fox Tennis team. Tina wins 70% of her matches and Brenda wins 55% of her matches. Assuming independence, determine the probability that Brenda wins her next match and Tina does not

$P(B) = 0.55$

$P(\bar{T}) = 1 - 0.7$

$= 0.3.$

$P(B \& \bar{T}) = 0.55 \times 0.3$

$= 0.165.$

5×11 $\quad 15 \times 11 = 165$

7. The probability that you are late for your bus is 7/11. The probability that the bus arrives on time is 1/11. What is the probability that you are late and the bus is also late?

$P(\text{LATE}) = \frac{7}{11}$

$P(\text{BUS LATE}) = 10/11$

$P(\text{LATE} \& \text{BUS LATE}) = \frac{7}{11} \times \frac{10}{11}$

$= \frac{70}{121}$

8. A biased coin with P(heads)=0.6 was tossed 4 times. The coin came up heads 3 times, 3 heads first then tails. What is the probability of this outcome?

$P(H) = 0.6$

$P(T) = 0.4$

$P(HHHT) = 0.6 \times 0.6 \times 0.6 \times 0.4$

$= 0.216 \times 0.4$

$= 0.0864$

9. In 6 tosses of a coin, the first ~~two~~ THREE tosses resulted in 3 heads. What is the probability that the 6 tosses will produce 4 heads? (ans:3/8)

HHH __ __ __

HHH TTT

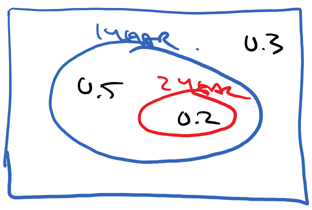
HHT TTH

HTH THT

THH HTT

$\frac{3}{8}$

10. The probability that your car battery will last one year is 0.7 and that it will last two years is 0.2. At the end of the first year, what is the probability that the battery will last until the end of the second year?



- PROB THAT IT IS IN THE RED CIRCLE WHEN WE KNOW IT MUST BE IN THE BLUE CIRCLE

$P(x > 2) = \frac{2}{7}.$

11. There are 3 printers in a school and the probability that printer A is broken is 1/4, printer B is broken is ½, and printer C is broken is 1/7. What is the probability of each event?

i) All 3 machines are broken? Ii) only two machines are broken?

$\frac{1}{4}\times\frac{1}{2}\times\frac{1}{7}=\frac{1}{56}$ $\quad$ $\frac{1}{4}\times\frac{1}{2}\times\frac{6}{7}+\frac{3}{4}\times\frac{1}{2}\times\frac{1}{7}\times\frac{1}{4}\times\frac{1}{2}\times\frac{1}{7}$

12. Two hockey teams "A" and "B" have a shootout to see which team will win the gold medal. The teams take turns to attempt to score a goal. The first team to score a goal wins. The probability that team A will score a goal is 0.30. The probability that team B will score a goal is 0.2. What is the probability of each of the following events?

a. Team "A" shoots first and then team "B" wins on the next shot

$\bar{A}\times B$

$0.7\times 0.2=0.14$.

b. Team "A" shoots first and then scores on its fifth shot

$\bar{A}\bar{B}\times\bar{A}\bar{B}\times\bar{A}\bar{B}\times\bar{A}\bar{B}\times A=(0.7\times 0.8)^4\times 0.3$

$=$

c. Team "B" shoots first and team "A" scores on its 3rd shot

$\bar{B}\bar{A}\times\bar{B}\bar{A}\times\bar{B}\times A=(0.8\times 0.7)^2\times 0.8\times 0.3$

d. Team "B" shoots first and team "A" wins

INFINITE GEOMETRIC SUM

$a=0.24 \quad r=0.56$

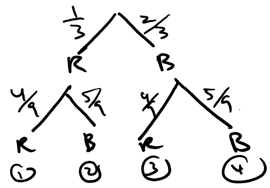
$(\bar{B}A)+(\bar{B}\bar{A})\bar{B}A+(\bar{B}\bar{A})(\bar{B}\bar{A})\bar{B}A+(\bar{B}\bar{A})(\bar{B}\bar{A})(\bar{B}\bar{A})\cdot\bar{B}\cdot A+\ldots$

$0.8\times 0.3+(0.8\cdot 0.7)\,0.8\times 0.3+(0.8\times 0.7)^2\,0.8\times 0.3+(0.8\times 0.7)^3\times 0.8\times 0.3+(0.8\times 0.7)^4(0.8)(0.3)+\ldots=\frac{0.24}{1-0.56}$

$=$

13. In bag A there are 2 red balls and 4 black balls. In bag B there are 4 red balls and 5 black balls. One ball is drawn from each bag. What is the probability of each event?

A: 2R 4B $\quad$ B: 4R 5B.



a. Drawing two balls with different colours

2 + 3

$\frac{1}{3}\times\frac{5}{9}+\frac{2}{3}\times\frac{4}{9}$

$=\frac{5}{27}+\frac{8}{27}=13/27$.

b. Drawing two ball with the same colours 1 + 4

$\frac{1}{3}\times\frac{4}{9}+\frac{2}{3}\times\frac{5}{9}$

$=\frac{4+10}{27}=14/27$.

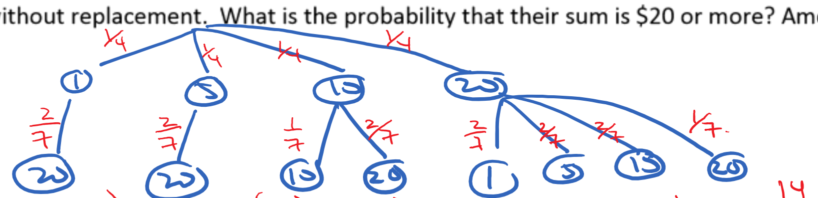
14. Juan rolls a fair regular octahedral die marked with numbers 1 through 8. Then Amos rolls a fair six sided die. What is the probability that the product of the two rolls is a multiple of 3? Amc 12 2002

	1	2	3	4	5	6	7	8
1			✓			✓		
2			✓			✓		
3	✓	✓	✓	✓	✓	✓	✓	✓
4			✓			✓		
5			✓			✓		
6	✓	✓	✓	✓	✓	✓	✓	✓

$P(A\text{ or }B)=\frac{24}{48}$

$=1/2$.

15. An envelope contains eight bills: 2 ones, 2 fives, 2 tens, and 2 twenties. Two bills are drawn at random without replacement. What is the probability that their sum is $20 or more? Amc 12 2005



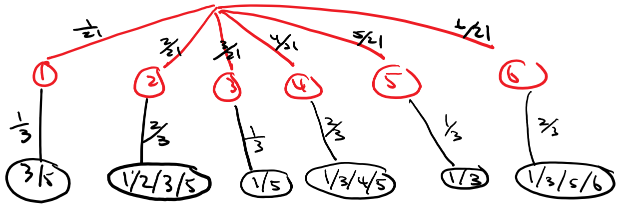
$\left(\frac{2}{28}\right)+\left(\frac{2}{28}\right)+\left(\frac{1}{28}\right)+\left(\frac{2}{28}\right)+\left(\frac{2}{28}\right)+\left(\frac{2}{28}\right)+\left(\frac{2}{28}\right)+\frac{1}{28}=\frac{14}{28}$

$=1/2$

16. On a standard die one of the dots is removed at random with each dot equally likely to be chosen. The die is then rolled. What is the probability that the top face has an odd number of dots? AMC 12 2005

16. On a standard die one of the dots is removed at random with each dot equally likely to be chosen. The die is then rolled. What is the probability that the top face has an odd number of dots? AMC 12 2005

Dot	P(d)
•	1/21
••	2/21
•••	3/21
••••	4/21
•••••	5/21
••••••	6/21

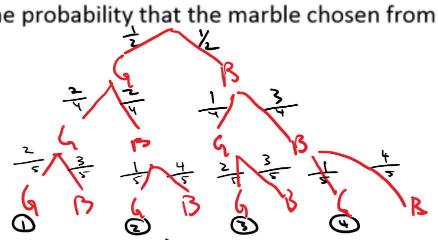


$\frac{1}{63} + \frac{4}{63} + \frac{3}{63} + \frac{8}{63} + \frac{5}{63} + \frac{12}{63}$

$= \frac{33}{63} = \frac{11}{21}$

17. Box 1 contains one gold marble and one black marble. Box 2 contains one gold marble and two black marbles. Box 3 contains one gold marble and three black marbles. Whenever a marble is chosen randomly from one of the boxes, each marble in that box is equally likely to be chosen. A marble is randomly chosen from Box 1 and placed in Box 2. Then a marble is randomly chosen from Box 2 and placed in Box 3. Finally a marble is chosen from Box 3. What is the probability that the marble chosen from Box 3 is gold?

B1: 1G 1B   B2: 1G 2B   B3: 1G 3B



$\left(\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{2}{5}\right) + \left(\frac{1}{2}\cdot\frac{2}{4}\cdot\frac{1}{5}\right) + \left(\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{2}{5}\right) + \left(\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{5}\right)$

$\frac{1}{10} + \frac{1}{20} + \frac{1}{20} + \frac{3}{40} = \frac{11}{40}$

18. Amy and Bill alternately toss a fair die with Amy going first. A neutral third party keeps a running tab of the combined sum of all their throws. Whoever reaches a combined sum divisible by 6 wins. What is the probability that Amy wins?

$P(A) = 1/6 \quad P(B) = 1/6$

$P(\bar{A}) = 5/6 \quad P(\bar{B}) = 5/6$

$A + \bar{A}\bar{B}\cdot A + (\bar{A}\bar{B})^2\cdot A + (\bar{A}\bar{B})^3\cdot A + \cdots$

Geometric sum $\frac{a}{1-r}$

$\frac{1}{6} + \left(\frac{25}{36}\right)\left(\frac{1}{6}\right) + \left(\frac{25}{36}\right)^2\cdot\frac{1}{6} + \left(\frac{25}{36}\right)^3\left(\frac{1}{6}\right) + \cdots$

$= \frac{1/6}{1-\frac{25}{36}} = \frac{6}{36-25} = \frac{6}{11}$

19. Each of the 2018 boxes in a line contains a single red marble, and for $1 \le k \le 2018$, the box in the k[th] position contains "k" white marbles. Isabella begins at the first box and successively draws a single marble at random from each box, in order. She stops when she first draws a red marble. Let P(n) be the probability that Isabella stops after drawing exactly "n" marbles. What is the smallest value of "n" for which $P(n) < \frac{1}{2018}$. AMC 10

1R 1B | 1R 2B | 1R 3B | 1R 4B

$P(1) = \frac{1}{2} = \frac{1}{1\times 2}$

$P(2) = \frac{1}{2}\times\left(\frac{1}{3}\right) = \frac{1}{2\times 3}$

$P(3) = \frac{1}{2}\times\frac{2}{3}\times\frac{1}{4} = \frac{1}{3\times 4}$

$P(4) = \frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{1}{5} = \frac{1}{4\times 5}$

$\therefore P(n) = \frac{1}{n}\times\frac{1}{n+1} < \frac{1}{2018}$

$\frac{1}{45}\times\frac{1}{46} < \frac{1}{2018}$

44×45

2025 − 45

$\boxed{n=45}$

20. Jason has 3 green baskets, 3 red baskets, 3 blue baskets, and 3 yellow baskets. He randomly distributes 4 eggs among the green baskets, with each egg equally likely to be put in each basket. Similarly, he distributes 3 eggs among the red buckets, 2 eggs among the blue baskets, and 1 egg among the 3 yellow baskets. Once he is finished, what is the probability that a green basket contains more eggs that each of the other 11 baskets? $\boxed{\text{Fermat 2018}}$

(3) Green (4 Eggs)

Red (3 Eggs)

Blue (2 Eggs)

Yellow (1 Egg)

Green: 4,0,0 $\frac{1}{27}$ A, B, C

3,1,0 $\frac{24}{81}$

2,2,0 $\frac{18}{81}$

2,1,1 $\frac{36}{81}$

A B C A B C A B C A B C = 81

AA BB   AABC −12

AACC   BBAC

BBCC   CCAB

Red: 3,0,0

2,1,0 $\frac{18}{27}$

1,1,1 $\frac{6}{27}$

$\frac{1}{27}$ $\frac{24}{81}$ $\frac{36}{81}$

2,1,1,0 ... 1,2,1

1,1,1,1 ... 6/27

A, B, C,

A B C — A B C — A B C ... 27

AAB BBA CCA

AAC BBC CCB

A, B, C →

BLUE:

A B C

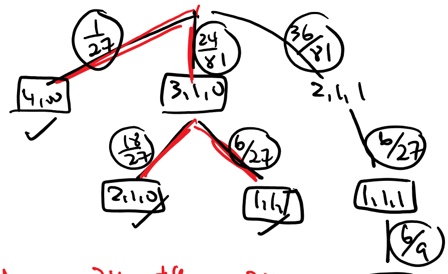
2 0 0

1 1 0

A B C — A B C ... 6/9

AB BC

AC



$$\frac{1}{27} + \frac{24}{81} \times \frac{18}{27} + \frac{24}{81} \times \frac{6}{27} + \frac{36}{81} \times \frac{6}{27} \times \frac{6}{9} = \frac{89}{243}$$